APPLIED MATHEMATICS REVISION QUESTIONS

STATISTICS.

1. Consider the following data

CI	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 - 39
F	2	3	6	2	4	1

i) Draw the histogram of the data

ii) Determine the mode from your histogram or otherwise

iii) Calculate the mean distribution using the assumed formula

2. The table below shows the marks obtained by students of mathematics in a certain school.

Marks	Number of students
30 - < 40	2
40 - < 50	15
50 - < 55	10
55 - < 60	11
60 - < 70	30
70 - < 90	29
90 - < 100	3

i) Calculate the mean, median and standard deviation

ii) Draw a histogram for the data, hence determine the modal mark

iii) Draw an ogive, hence determine the median from the ogive and compare it with the calculated value.

3. Consider the following data below

137	140	150	140	157	131
141	142	162	169	166	129
138	170	170	152	161	139
122	131	139	170	165	145
140	147	125	134	153	145
151	128	129	121	154	167
122	165	128	149	140	136
130	170	133	139	136	150

Construct a frequency table with classes of equal width starting with 121 – 130 and use it to calculate the mean, mode and standard deviation

4. Consider the following data

CI	10 – 14	15 – 19	20 – 29	30 – 34	35 – 44	45 - 49
f	2	3	6	2	4	1

i) Draw the histogram of the data

ii) Determine the mode from your histogram

iii) Calculate the quartile deviation

5. a) In 1995 the prices of commodities A, B and C were shs. 720, shs. 830 and shs. 950 respectively. Given that the prices after 5 years were shs. 860, shs. 940 and shs. x and the simple aggregate price index number was 140. Find x

b) The price of a radio in 1995 was shs. 30,000. The index number for the price of a radio in 1985 was 1.6 based on 1975. 1n 1995, it was 0.75 based on 1985. Calculate

i) The prices of the radio in 1975 and 1985

ii) The price relative in 1995 based on 1975

6. A pharmacist had the following record of unit price and quantities of drugs sold for the years 1990 and 1991

Drug	Quantities in cartons		Unit price per carton	
	1990	1991	1990	1991
Aspirin	40	45	80	125
Panadol	70	90	100	90
Quinine	08	10	55	70
flue caps	10	10	90	100

Using 1990 as the base year, calculate

i) The price relative for each drug

ii) The simple aggregate price index number for 1991

iii) Fisher's index number for 1991

7. The table below shows the performance of 100 students in a resource examination

Score (%)	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49	50 - 59
Candidates	10	x	25	30	y	10

i) Given the median is 30.5, determine the values of x and y

ii) Hence calculate the average score and the mode

iii) Construct a cumulative frequency curve and use it to estimate the inter-quartile range

8. Using the figures in the table below

Food	2000		2005	
	Quantity in kg	Price per kg(shs)	Quantity in kg	Price per kg(shs)
Maize	20	650	25	700
Wheat	10	1500	8	1600
Beans	5	150	8	200

Calculate

i) Paasche aggregate price index

ii) Laspeyre aggregate price index

9. The cost of making a cake is calculated from the baking flour, sugar, milk and eggs. The table below gives the cost of these items in 1990 and 2000

Item	1990	2000	Weight, w
Flour per kg	600	780	12
Sugar per kg	500	400	5
Milk per litre	250	300	2
Eggs per egg	100	150	1

Using 1990 as the base year, calculate

i) The price relative for each item. Hence find simple price index for the cost of making a cake

ii) The simple aggregate price index number for 2000

iii) Fisher's index number for 2000

10. The following items are used in 1990 and 1985 as shown in the table below

Item	1985 price (£)	1990 price (£)	Weight, w
Radio	56	60	4
Shoes	45	50	2
Cap	15	20	1

Calculate the weighted price index number using 1985 as a base year

11. Given that A and B are mutually exclusive events such that P(A) = 0.3 and P(AUB) = 0.7. Find

i) P(B) ii) $P(A^1 \cap B)$

iii) $P(A^1 U B)$ iv) $P(A^1 \cap B^1)$

12. Given two events A and B such that $P(A^1) = {}^2/_3$, $P(B) = {}^1/_2$, and $P(A \cap B) = {}^1/_{12}$. Find

i) P(A) ii) P(AUB)

iii) $P(A^1 \cap B)$ iv) $P(A^1 U B^1)$

13. Events A and B are such that $P(A) = {}^1/_2$, $P(B) = {}^3/_8$, and $P({}^A/_B) = {}^7/_{12}$. Find

i) $P(A \cap B)$ ii) $P({}^B/_A)$

iii) $P(A^1 \cap B)$ iv) $P\left({}^B/_{A^1}\right)$

14. Two independent events A and B are such that P(A) = 0.4, P(B) = b and P(AUB) = 0.7. Find

i) The value of b ii) $P(A \cap B)$

iii) $P(A \cap B^1)$ iv) $P(A \, U \, B^1)$

15. There are 3 black and 2 white balls in each of two bags. A ball is taken from the first bag and put in the second bag, and then a ball is taken from the second bag into the first bag. What is the probability that there are now the same numbers of black and white balls in each bag as there were to begin with?

16. A fair coin is tossed four times. Use the tree diagram to calculate the probability of obtaining

    i) Exactly three heads

    ii) At least two tails

    iii) Exactly one head

    iv) Not more than three tails

17. The probabilities that a man makes a journey by car, air and road are respectively $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$. If the probabilities of an accident occurring when he uses these means of transport are $\frac{1}{3}, \frac{3}{10}$ and $\frac{1}{10}$ respectively

    i) What is the probability of an accident occurring

    ii) If the accident is known to have happened, find the probability that the man was travelling by Air

    ii) If it is known that the man reached safely, find the probability that he used the road

18. Three workers of dairy cooperation, Joy, Jane and Juliet seal milk sackets. On a particular day Joy seals 48%, Jane 30% and Juliet 22%, the probability that Joy seals wrongly is 0.53, Jane seals wrongly is 0.30 and Juliet seals wrongly is 0.17. Find the probability that a sacket was sealed wrongly and a wrongly sealed sacket found by the checker was sealed by Joy.

19. A bag contains white, yellow and blue beads in the ratio of 3: 4: 1. Two beads are selected at random without replacement, one after the other, obtain the probability that

    i) Two of the selected beads are of the same colour

    ii) The selected beads of different colours

20. A committee of 5 is selected from 4 men and 3 women.

    i) In how many ways can this be done if there must be more men than women?

    ii) What is the probability that the committee consists of 2 men

    iii) A woman is selected at random, find the probability that she belongs to this committee.

21. A discrete random variable X has a pdf, f(x) given by

$$f(x) = \begin{cases} k(x+1), & x = 1, 2, 3, 4. \\ kx, & x = 5, 6, 7 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) $P(2 \leq x < 7)$

iii) The mean and mode of X

iv) The standard deviation of X

v) The semi inter-quartile range

22. A random variable X has the following cdf, F(x) given by

$$F(x) = \begin{cases} \frac{kx}{4}, & x = 3, 4, 5 \\ k(x-3), & x = 6, 7, 8 \\ 1, & x \geq 8 \end{cases}$$

Find

i) The value of the constant, k

ii) The pmf of x

iii) Expectation of X

iv) $P(x \geq E(x))$

v) Median and variance of X

23. The number of days the machine breaks down in a week follows a discrete random variable X with the following pdf, f(x) given by

$$f(x) = \begin{cases} kx^2, & x = 1, \ldots, 4. \\ k(7-x)^2, & x = 5, 6 \\ \frac{k}{7}, & x = 7 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) The mean number of days the machine breaks down in a week

iii) The probability that the machine breaks down not more than 3 days in a week

24. A discrete random variable X has a probability density function, f(x) given by

$$f(x) = \begin{cases} \frac{x}{k}, & x = 1, 2, \ldots, n \\ 0, & \text{elsewhere} \end{cases}$$

Given that the expectation of X is 3. Find

i) The value of n and the constant, k

ii) The median of X

iii) The variance of X

iv) $P(x = 2/x \geq 2)$

v) The cdf of X

25. The number of days the machine breaks down in a month follows a discrete random variable X with the following pdf, f(x) given by

$$f(x) = \begin{cases} k\left(\frac{1}{4}\right)^x, & x = 0, 1, 2, \ldots \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of a constant, k

ii) The probability that the machine breaks down not more than 2 times in a month

26. a) The probability distribution of a discrete random variable X is given by

$$P(X = r) = \begin{cases} kr, & r = 1, 2, 3, \ldots, n \\ 0, & \text{elsewhere} \end{cases}$$

i) Show that the constant, $k = \frac{2}{n(n+1)}$

ii) Find in terms of n the mean and variance of X

b) The discrete random variable X has a probability density function given by

$$f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3, 4, 5 \\ C, & x = 6 \\ 0, & \text{otherwise} \end{cases}$$

Determine

i) The value of the constant, C

ii) The mean of X

iii) The mode of X

27. A random variable X has the following probability distribution

$P(X = -2) = P(X = -1) = 2P(X= 0)$, $P(X = 1) = 0.2$, $2P(X = 2) = P(X =3)$

The mean of X equals the probability with which X assumes the value of -1. Find

i) $P(X = 2)$ and $P(X = 0)$

ii) $P(x \leq 2/x \geq 0)$

iii) The standard deviation of X

iv) The upper quartile

28. The random variable X takes integer values only and has a pdf given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ k(10 - x), & x = 6, 7, 8, 9 \\ 0, & \text{elsewhere} \end{cases}$$

Find

i) The value of the constant, k

ii) $E(X)$ and $Var(X)$

iii) $E(2X - 3)$ and $Var(2X - 3)$

iv) $P(2 \leq 2X - 2 \leq 11)$

29. Two random variables X and Y take on integer values with probabilities as given below

X	2	3	4	5	6
P(X = x)	0.25	0.15	0.10	0.30	0.20

Y	-1	0	1	2	3
P(Y = y)	0.20	0.25	0.40	0.10	0.05

Find;

i) $E(2X - 4)$

ii) $Var(2X + Y)$

iii) $Var(3X - 4Y)$

iv) $Var(X + Y)$

30. A discrete random variable X has the following distribution given by

X	1	2	3	4	5
P(X = x)	$\frac{4}{15}$	$\frac{3}{40}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{11}{24}$

Two other random variables Y and Z are defined in terms of X as follows

Y = 2X + 1 and Z = 3X – 2

Find

i) The pmf of Y and Z ii) Standard deviation of Z

iii) The variance of Y iv) Draw the graph of cdf of Z

v) P(3 < Z < 9)

vi) Plot a graph of pmf of Y use it to find the mode

31. A random variable X has pdf, f(x) where

$$f(x) = \begin{cases} k[2 - (x+1)^2], & 1 \leq x \leq 3 \\ 0, & \text{elsehere} \end{cases}$$

Determine the

i) The value of a constant

ii) The c.d.f, F(x) of X

32. A random variable X has p.d.f f(x) given by

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ 2k\,(x-1)^2, & 2 \leq x \leq 5 \\ 0, & \text{elesewhere} \end{cases}$$

Determine the

i) Value of a constant, k ii) mean of X

iii) Standard deviation of X iv) $P(|x - 3| < 1)$

33. A random variable X has pdf f(x) given by

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 1 \\ \frac{k}{2}(x+1), & 1 \leq x \leq 3 \\ 2k, & 3 \leq x \leq 4 \\ 0, & \text{elesewhere} \end{cases}$$

i) Sketch a graph of f(x) and use it to find ii) The constant, k

iii) Calculate the cdf, F(x) of X iv) P(1 ≤ x ≤ 4)

v) Calculate the mean and standard deviation of x

34. A random variable X has the following cdf, F(x) given by

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{k}{2}x^2, & 0 \leq x \leq 1 \\ \frac{k}{2} + \frac{k}{4}(6x - x^2 - 5), & 1 \leq x \leq 3 \\ 1, & x \geq 3 \end{cases}$$

Find the

i) Value of the constant, k ii) The pdf, f(x) of X

iii) $P(1 \leq 2x \leq 3)$ iv) Sketch the graph of f(x)

v) $P(0 \leq x \leq 2/0.5 \geq x)$

35. A random variable X has probability density function, f(x) given by

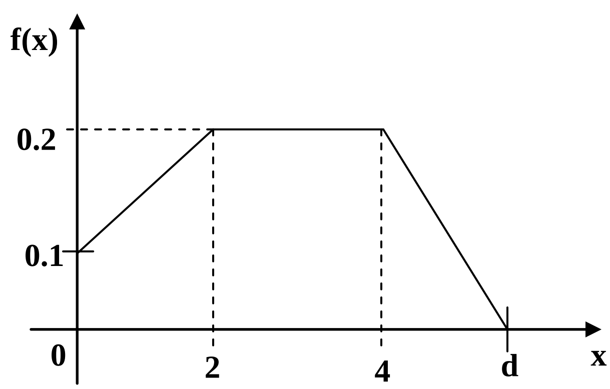
$$f(x) = \begin{cases} \frac{2}{3}a(x + a), & -a \leq x \leq 0 \\ \frac{1}{3a}(2a - x), & 0 \leq x \leq 2a \\ 0, & \text{elesewhere} \end{cases}$$

Determine the

i) Value of the constant, a ii) median of X iii) $P(x \leq 1.5/x > 0)$

iv) Cumulative distribution function, F(x) and sketch it

36. The motion of a motorist may be modeled into a continuous random variable X with the graph of f(x) as shown below;



Calculate the

i) value of d ii) pdf, f(x) of X

iii) standard deviation iv) cdf, F(x) of X

v) the median of X

37. A continuous random variable X has a pdf given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k, & 1 \le x \le 2 \\ 2(3-x), & 2 \le x \le 3 \\ 0, & \text{elesewhere} \end{cases}$$

i) Sketch f(x) and use it to find the value of k

ii) Calculate the mean of X iii) Find $P(x < 2.5)$

iv) Determine the cumulative distribution function, F(x)

38. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} \frac{1}{3}(x-2), & 2 \le x \le 3 \\ a, & 3 \le x \le 5 \\ 2 - bx, & 5 \le x \le 6 \\ 0, & \text{elesewhere} \end{cases}$$

i) Find the value of a and b

ii) Determine $P(x > \frac{11}{2})$

iii) Determine the inter-quartile range

iv) Obtain the expression for F(x)

39. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} kx(3-x), & 0 \le x \le 2 \\ k(4-x), & 2 \le x \le 4 \\ 0, & \text{elesewhere} \end{cases}$$

Find

i) The value of k iii) The mean of X

ii) $P(1 < x < 3)$

iii) Cumulative distribution function, F(x)

40. A continuous random variable X has a probability function, f(x) given by

$$f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ k(4-x^2), & 1 \le x \le 2 \\ 0, & \text{elesewhere} \end{cases}$$

Find

i) The value of k

ii) E(X) and Var (x)

iii) Cumulative distribution function, F(x)

41. A continuous rectangular random variable X has limits 3 and 8

i) Draw the graph of its pdf

ii) Use the graph to find $P(x > 5)$

iii) Find the cdf and draw its graph

iv) Find the median of X

42. The continuous random variable X has the pdf, f(x) given by

$$f(x) = \begin{cases} \frac{1}{k}, & 24 \le x \le 34 \\ 0, & Otherwise \end{cases}$$

Find the

i) Value of k

ii) Mean, median and quartile deviation

iii) Cdf of X

iv) Value of a if $P(x > a) = 0.65$

43. It is known that a continuous random variable X has uniform distribution over the interval [20, 30]. Find the

i) Mean and standard deviation of X

ii) Median and quartile deviation of X

iii) $P(25 < x < 29/x > 27)$

iv) cdf of X

44. A continuous random variable X has uniform distribution over the interval [d, e]. d < e. Show that

i) The mean is equal to the median

ii) $SD(x) = \frac{e-d}{2\sqrt{3}}$

iii) If $E(x) = 3$ and $Var(x) = \frac{4}{3}$ then $e = 5$ and $d = 1$

45. A discrete random variable X has uniform distribution over the interval [a, b].

a < b, a and b are integers

i) Show that its pdf satisfies properties of the pdf of a discrete random variable

ii) Prove that its mean is ½ (a + b + 1)

iii) Obtain the expression for the variance hence its SD

iv) Find its median and quartile deviation if a = 10 and b = 25

MECHANICS

1. A train approaching a station covers successive half kilometers in 16s and 20s respectively. Assuming the retardation to be uniform, find the further distance the train run before coming to a stop.
2. Two points P and Q are x m apart. A boy starts from rest at P and moves directly towards Q with an acceleration of a ms^{-2} until he acquires a speed of Vms^{-1}. He maintains this speed for T seconds and then brought to rest at Q under a retardation of a ms^{-2}. Prove that $T = \frac{x}{V} - \frac{V}{a}$
3. A car started from rest accelerated uniformly for 2 minutes and then maintained a speed of $50kmh^{-1}$. Another car started 2 minutes later from the same spot and this car too accelerated uniformly for 2 minutes and it then maintained a speed of $75kmh^{-1}$.

   i) Draw a velocity – time graph and find when and where the second car overtook the first.

   ii) The car maintained the speed of $50kmh^{-1}$ for 10minutes. It then decelerated uniformly for further 2½ minutes before coming to rest. How far has the car travelled from the start?
4. A motor car A passes a certain point P with a speed of $7.5ms^{-1}$ and an acceleration of $0.3ms^{-2}$. Five seconds later a car B passes P with a speed of $15.0ms^{-1}$ and an acceleration of $0.2ms^{-2}$. Prove that if their maximum speed is $30.0ms^{-1}$, B will ultimately be travelling 131m a head of A.
5. A car starting from rest is uniformly accelerated during the first 0.5km of its run, then run 1.5km at a uniform speed and is afterwards brought to rest in ¼km under uniform retardation. If the time for the whole journey is 5minutes, find the uniform acceleration and uniform retardation.
6. A particle starts from rest and moves in a straight line with uniform acceleration. In 4 seconds of its motion it travels 12m and in the next 5 seconds it travels 30m. Find its

   i) Acceleration

   ii) Final velocity

7. A train travels a long a straight track between two stations A and B. the train starts from rest at A and accelerates at $1.25ms^{-2}$ until it reaches a speed $20ms^{-1}$. It then travels at this speed for a distance of 1.56km and then decelerates at $2ms^{-2}$ to come rest at B.
   i) Sketch a velocity-time graph for the motion of the train
   ii) Find the distance from A to B
   iii) Find the total time of the journey
8. A, B and C are three points which lie in that order on a straight road with AB = 95m and BC = 80m. a car is travelling along the road in the direction ABC with constant acceleration of ams^{-2}. The car passes through A with speed u ms^{-1} reaches B later and C two seconds after that. Find the values of a and u.
9. Two stations A and B are a distance of 6x mtres apart along a straight line. A train starts from rest at A and accelerates uniformly to a speed of Vms^{-1} covering a distance of x metres. The train then maintains this speed until it has travelled a further 3x metres. It then retards uniformly to rest at B. sketch a velocity- time graph for the motion of the car and show that if T is the time taken to travel from A to B then T = $\frac{9x}{V}$ seconds.
10. P, Q and R are points on a straight road such that PQ = 20m and QR = 55m. a cyclist moving with uniform acceleration passes P and then notices that it takes him 10s and 15s to travel between (P and Q) and (Q and R) respectively. Find his uniform acceleration.
11. A body is projected vertically upwards with a velocity of $25ms^{-1}$. Find
    i) How high it will go
    ii) What times elapses before it is at a height of 20m
    iii) The time to reach maximum height hence time of flight
12. A ball is thrown vertically upwards with a speed of $42ms^{-1}$. If it falls past the point of projection into a sea of depth 80m, find when it strikes the bottom of the sea.

13. A particle is projected from a point O with an initial velocity 3i + 4j. Find in vector form the velocity and position of the particle at any time, t.

14. A stone is thrown vertically upwards with a velocity of $25ms^{-1}$, if another stone is thrown vertically upwards 4 seconds later with the same speed from the same point of projection. Determine when and where the two stones meet.

15. A stone is dropped from the top of the building and at the same instant another stone is thrown vertically upwards from the bottom of the building with a speed of $20ms^{-1}$. They pass each other three seconds later. Find the height of the building.

16. A particle is projected vertically upwards with a certain velocity and it is found that when it is 400m from the ground it takes 8 seconds to return to the same point again. Find the velocity of projection and the time of flight.

17. The ball is thrown vertically down wards from the top of the tower and has an initial speed of $4ms^{-1}$. If the ball hits the ground 2 seconds later. Find
    i) the height of the tower
    ii) the speed at which the ball strikes the ground.

18. A stone is thrown vertically upwards from the ground level at a speed of $24.5ms^{-1}$. Find how long after projection the stone is at a height of 19.6m above the ground for the first time and the second time and how long is the stone at least 19.6m above the ground level.

19. A ball is thrown vertically upwards with a speed, u after time, t another ball is thrown vertically upwards from the same point with the same speed. Prove that they will meet after elapse of $\left(\frac{t}{2} + \frac{u}{g}\right)$ seconds from the time the first particle was projected hence show that the distance travelled is $\left(\frac{4u^2 - g^2t^2}{8g}\right)$ m.

20. A particle is projected vertically upwards from a point O with a speed $\frac{4}{3}$V. after it has travelled a distance $\frac{2}{5}$x above O on its upward motion, a second particle is projected vertically upwards from the same point and with the same initial speed. Given that the particles collide at a height $\frac{2}{5}$x above O, x and V being constant, show that

i) at maximum height H, $8V^2 = 9gH$

ii) when the particles collide $9x = 20H$

21. Find the angle between a and b, given that

a = 5i + 6j + k and b = 2i + j

22. A particle of mass 5kg at rest at appoint (1, -4, 4) is acted upon by three forces F_1 = 3i + 3j, F_2 = 2j + 4k and F_3 = 2i + 6k. Find

i) Acceleration of the particle

ii) Velocity and speed of the particle after 4 seconds

iii) Position and the distance of the particle after 4 seconds

23. The forces F_1 = 2i + 3j, F_2 = i + 3j and F_3 = i+ 2j act on a particle of mass 2kg located at (1, -1). Find

i) The magnitude and direction of the resultant force

ii) Position and the distance of the particle after 4 seconds

24. Three forces F_1 = 6i + 3j, F_2 = -2i + 3j and F_3 = λi act on a particle at the origin, if the magnitude of the resultant is 10N. Calculate the two possible values of λ and the two possible directions of the line of action of the resultant.

25. A particle of mass 3kg moving on the curve described by

r = 4sin3ti + 8cos3tj where r is the position vector at time, t.

i) Determine the position and the velocity of the particle at time, t = 0s

ii) Show that the force acting on the particle is –27r.

26. A particle of mass 2kg initially at rest at (0, 0, 0) is acted upon by the force $\begin{pmatrix} 2t \\ t \\ 3t \end{pmatrix}$ N. Find

i) Acceleration and velocity after 3 seconds.

ii) Distance moved after 3 seconds.

27. A particle of weight 8N is attached at a point B of a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at an angle of 30^o to the vertical. A force, F at B acting at right angles to AB keeps the particle in equilibrium. Find the magnitude of F and the tension in the string.

28. A particle of mass 3kg is attached to the lower end B of an inextensible string. The upper end A of the string is fixed to a point on the ceiling of a roof. A horizontal force of 22N and an upward vertical force of 4.9N act upon the particle making it be in equilibrium with the string making an angle, $\propto$ with the vertical. Find the value of $\propto$ and the tension in the string.

29. A non-uniform rod of mass 9kg rests horizontally in equilibrium supported by two light inextensible strings tied to the ends of the rod. The strings make angles of 50^o and 60^o with the rod. Calculate the tensions in the strings.

30. A particle moving with an acceleration given by $a = 4e^{-3t}i + 12\sin tj - 7\cos tk$ is located at a point (5, -6, 2) and has velocity $v = 11i - 8j + 3k$ at time $t = 0$. Find the

i) Magnitude of the acceleration when $t = 0$

ii) Velocity at any time, t

iii) Displacement at any time, t

31. A particle with position vector $10i + 3j + 5j$ moves with constant speed of $6ms^{-1}$ in the direction $i + 2j + 2k$. Find its distance from the origin after 5 seconds.

32. A particle of mass 2 units moves under the action of force which depends on time, t given by $F = 24t^2i + (36t - 16)j$. given that t = 0, the particle is located at 3i – j and has a velocity 6i + 15j. Find

i) the kinetic energy of the particle at t = 2

ii) the impulse in moving the particle from t = 1 to t = 2 s

33. If the a = 6sin 6ti + 9cos 3t j. find displacement when t = ⅓▯ given that v = i + 3j and s = 5i + 2j when $t = \frac{\pi}{6}$

34. An object of mass 5kg is initially at rest at a point position vector is – 2i + j. if it is acted upon by a force F = 2i + 3j – 4k. Find

i) the acceleration

ii) the velocity after 3 seconds

iii) the distance from the origin after 3 seconds.

35. A particle of mass 2m rests on a rough plane inclined at an angle of $\tan^{-1}(3\mu)$ where μ is the coefficient of friction between the particle and the plane. The plane is acted upon by a force of PN

a) Given that the force acts along the line of greatest slope and that the particle is on a point of sliding up. Show that the maximum force possible to maintain the particle in equilibrium is

$$P_{max} = \frac{8\mu mg}{\sqrt{1 + 9\mu^2}}$$

b) Given that the force acts horizontally in a vertical plane through the line of greatest slope and that the particle is on a point of sliding down the plane. Show that the force required to maintain the particle in equilibrium is

$$P = \frac{4\mu mg}{1 + 3\mu^2}$$

36. A carton of mass 3kg rests on a rough plane inclined at an angle of 30^0 to the horizontal. The coefficient of friction between the carton and the plane is ⅓. Find a horizontal force that should be applied to make the carton just about to move up the plane.

37. A body of mass 8kg rests on a rough plane inclined at θ to the horizontal. If the coefficient of friction is μ, find the least horizontal force in terms of μ, θ and g which will hold the body in equilibrium.

38. A particle of mass 2kg rests on a rough inclined plane at an angle $\sin^{-1}\left(\frac{5}{13}\right)$. A force of 8N acts on the particle along the line of greatest slope

i) given that the particle is about to move up the plane, calculate the coefficient of friction between the particle and the plane

ii) if the 8N force is removed, find the acceleration of the particle down the plane.

39. A box of mass 2kg is at rest on a plane inclined at 25° to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane.

40. A particle of mass ½ kg is released from rest and slides down a rough plane inclined at 30° to the horizontal. It takes 6 seconds to go 3m.

i) find the coefficient of friction between the particle and the plane

ii) what minimum horizontal force is needed to prevent the particle from moving.

41. A particle of weight, W is at rest on an inclined plane under the action of a force, P acting parallel to the line of greatest slope of the plane in an upward direction. The angle of friction between the particle and the plane is λ and the angle of inclination of the plane to the horizontal is 2λ. Show that $P_{max} = W\tan\lambda(4\cos\lambda - 1)$ and $P_{min} = \mu W$ respectively. Calculate the coefficient of friction between the particle and the plane.

42. A particle of mass 2kg rests on a rough horizontal ground. The coefficient of friction between the particle and the ground is ½. Find the magnitude of the force, P acting upwards on the particle at an angle of $30^\circ$to the horizontal which will just move the particle.

43. A parcel of mass 2kg is place on a rough plane which is inclined at an angle of 45^o to the horizontal. The coefficient of friction between the parcel and the parcel and the plane is 0.25. Find the force that must be applied to the parcel in a direction parallel to the plane so that

i) the parcel is just prevented from sliding down the plane

ii) the parcel is just on the point of moving up the plane

iii) the parcel moves up the plane with an acceleration of $1.5ms^{-2}$

44. when at an angle of elevation, $\propto$ a gun fires a shot to hit a mark P on the horizontal plane, when the angle is reduced to 15^o the shot falls 100m short of P but when the elevation is 45^o it falls 400m beyond P. find the value of $\propto$ and distance of P from the gun.

45. The horizontal and vertical components of the initial velocity of a particle projected from a point O on the horizontal plane are p and q respectively.

i) Express the vertical distance Y travelled in terms of the horizontal distance X and the components of p and q.

ii) Find the greatest height, H attained and the range, R on the horizontal plane through O. hence show that $Y = \frac{4HX}{R^2}(R - X)$. Given that the particle passes through the point (20, 80) and H = 100m. Find the velocity of projection.

46. A particle P is projected from a point A with an initial velocity of $60ms^{-1}$ at an angle of 30^o to the horizontal. At the same time and the same instant a particle Q is projected in opposite direction with initial speed $50ms^{-1}$ from a point at the same level with A and 100m from A. given that the particles collide, find

i) the angle of projection of q

ii) the time when collision occurs

47. If T is the time of flight and x the horizontal range of a projectile, prove that $gT^2 = 2x \tan \propto$. Where $\propto$ is the angle of projection

48. A projectile having a horizontal range, R reaches a maximum height, H. prove that it must have been launched

i) with an initial speed equal to $\left[\frac{g(R^2+16H^2)}{8H}\right]^{½}$

ii) at an angle with the horizontal given by $\sin^{-1}\left[\frac{4H}{(R^2+16H^2)^{½}}\right]$

49. A ball is kicked from a point O so that it just clears two trees which are of height, h and at a distances x and y respectively from O. prove that if θ is the angle of projection of the ball,

i) $\tan\theta = \frac{h(x+y)}{xy}$

ii) the maximum height of the ball, $H = \frac{h(x+y)^2}{4xy}$

50. A particle is projected so as it just clears to walls each of height, h which lie at right angles with the plane of motion. The walls are at a distance, d apart and the first wall is at a distance, L from the point of projection. Show that the angle of elevation of the particle, ∝ is given by

$\tan\propto = \frac{h(2L+d)}{L(L+d)}$

51. A particle is projected from the top of the vertical cliff 160m high with a velocity of $180ms^{-1}$ at an angle of elevation of 30^{o}. Find the horizontal distance from the foot of the cliff to the point where it strikes the ground.

52. A particle is projected from a point height 3h above a horizontal plane, the direction of projection making an angle, ∝ with the horizon. Show that if the greatest height above the point of projection is h, the horizontal distance travelled before striking the plane is 6hcot ∝.

53. Two particles are projected simultaneously from the top and the bottom of a vertical cliff with angles of elevation ∝ and β respectively. They strike an object at the same point simultaneously. Show that if p is the horizontal distance of the object from the cliff, the height of the cliff is given by p(tanβ - tan∝)

54. Two boys stand on a horizontal ground at a distance, d apart. One throws a ball from a height, 2h with a velocity, V and the other catches it at a height, h above the horizontal at which the first boy throws the ball. Show that $gd^2 \tan^2\theta - 2V^2 d\tan\theta + gd^2 - 2V^2 h = 0$ holds if $d = 2h\sqrt{2}$ and $V^2 = 2gh$. Hence calculate

i) the value θ

ii) the greatest attained by the ball in terms of h, u, g and θ

55. A bullet is fired out with the initial velocity of projection is $240ms^{-1}$ to the sea in a horizontal direction from a gun situated on the top of a cliff 78.4m high. Calculate

i) the distance at which the bullet will strike the water from the foot of the cliff.

ii) the inclination to the horizontal at which the bullet will strike the surface of the water.

56. a) A particle is projected at an angle of elevation of 30^o with a speed of $21ms^{-1}$, if the point of projection is 5m above the horizontal ground. Find the horizontal distance that the particle travels before striking the ground.

b) A boy throws a ball at an initial speed of $40ms^{-1}$ at an angle of elevation, $\propto$. Show taking $g = 10ms^{-2}$, that the times of flight corresponding to a horizontal range of 80m are positive roots of the equation $T^4 - 64T^2 + 256 = 0$.

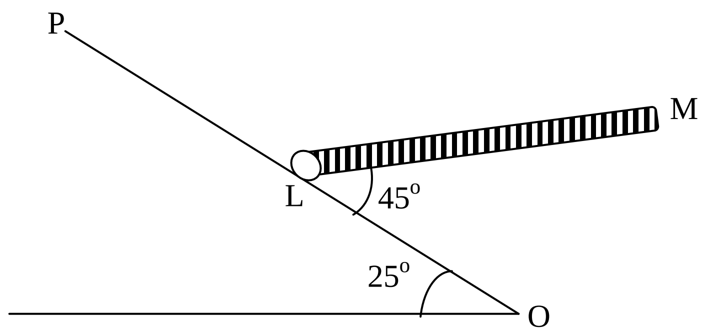
57. A particle is projected with a velocity of $40ms^{-1}$ at an angle of 60^o to the horizontal from the foot of the plane inclined at an angle of 30^o to the horizontal. Find the time at which the particle hits the plane.

58. A uniform ladder of length, 2L and weight, W rests in a vertical plane with one end against a rough vertical wall and the other end against a rough horizontal surface, the angles of friction at each end being $\tan^{-1}\frac{1}{3}$ and $\tan^{-1}\frac{1}{2}$ respectively.

i) if the ladder is in limiting equilibrium at either end. Find θ the angle inclination of the ladder to the horizontal.

ii) a man of weight 10 times that of the ladder begins to ascend it, how far will he climb before the ladder slips.

59. A uniform rod LM of weight, W rests with L on the smooth plane PO of inclination 25° as shown in the diagram below.



The angle between LM and the plane is 45°. What force parallel to PO applied at M will keep the rod in equilibrium? Give your answer in terms of W.

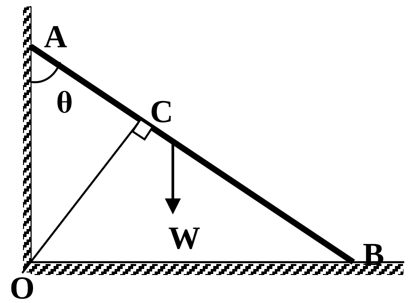
60. A non – uniform ladder AB, 10m long and mass 8kg lies in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a smooth vertical wall. If the Centre of gravity of the ladder is 3m from the foot of the ladder and the ladder makes an angle of 30° with the horizontal. Find the

i) coefficient of friction between the ladder and the ground

ii) reaction at the wall

61. Two smooth rods AB and AC each of weight, W and length 10cm are smoothly hinged at A. the ends B and C rest on a smooth horizontal plane. An extensible string joins B and C and the system is kept in equilibrium in a vertical plane with the string taut. An object of weight, 2W climbs the rod AC to a point E such that AE = 8cm. given that angle BAC is 2θ. Determine in terms of W and θ the reaction at the ends B and C and the tension in the string. Hence show that the reaction at the hinge A is given by $\frac{W}{10}\sqrt{49\tan^{-1}\theta + 4}$

62. The diagram below shows a uniform rod AB of weight, W and length, L resting at an angle, θ against a smooth vertical wall at A. The other end B rests on a smooth horizontal table. The rod is prevented from slipping by an inelastic string OC, C being a point on AB such that OC is perpendicular to AB and O on the point of intersection of the wall and the table. Angle AOB is 90°.

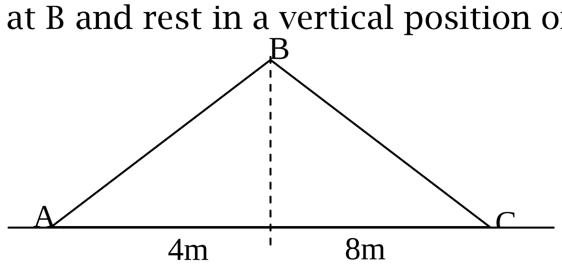


Find the

i) tension in the string

ii) reactions at A and B in terms of θ and W.

63. Two uniform rods AB and BC of masses 4kg and 6kg respectively are hinged at B and rest in a vertical position on the smooth floor as shown below.



A and C are connected by a rope

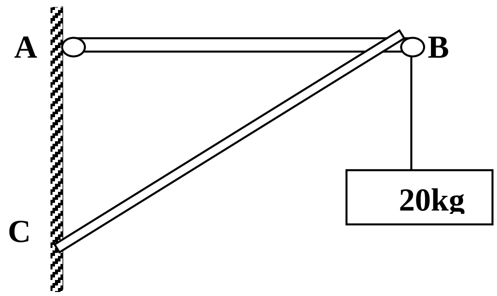
i) find the reactions between the rods and the floor at A and C when the rope is taut.

ii) if now a body is attached a quarter of the way up CB and the reactions are equal, find the mass of the body.

64. A non-uniform ladder AB is in equilibrium with A in contact with a horizontal floor and B in contact with a vertical wall. The ladder is in a vertical plane perpendicular to the wall. The Centre of gravity of the ladder is at G where AG = $\frac{2}{3}$AB. The coefficient of friction between the ladder and the wall is twice that between the ladder and the floor. If the

ladder makes an angle, θ with the wall and the angle of friction between the ladder and the floor is λ, prove that $4\tan\theta = 3\tan 2\lambda$. How far can a man of mass, m ascend the ladder without the ladder slipping given that $\theta = 45^o$ and the coefficient of friction between the ladder and the floor is ½.

65. The diagram below shows a uniform horizontal plank AB of length 3m and mass 2kg hinged to a vertical wall at A and supported at B by a light rod CB hinged to the same wall at C such that AC = 4m



If a mass of 20kg hangs from B, find the

i) tension force in the rod CB

ii) the magnitude of the reaction at A.

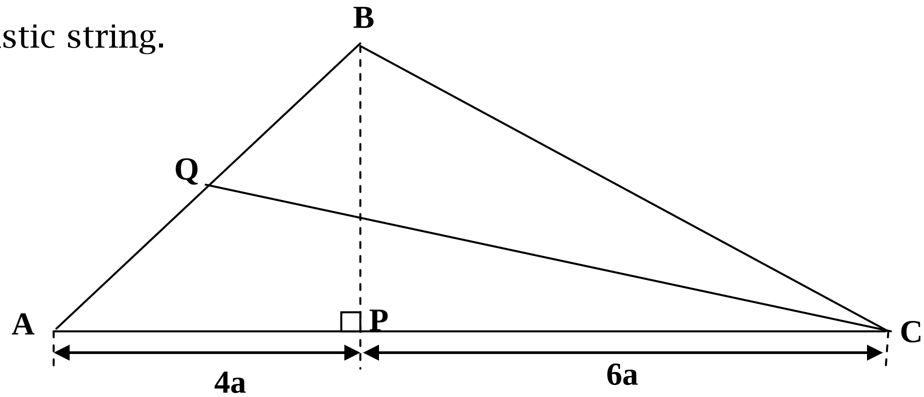
66. Two uniform rods AB and BC of equal length but of mass, M and 3M respectively are freely jointed together at B. the rods stand in a vertical plane with the ends A and C on a horizontal ground. The coefficient of friction, μ at the points of contact with the ground is the same and the rods are inclined at 60^o to each other. Given that one of the rods is on the point of sliding, find μ and the reaction at the hinge B when the rods are in this position.

67. A rod AB of length 0.6m long and mass 10kg is hinged at A. its centre of mass is 0.5m from A. a light inextensible string attached at B passes over a fixed smooth pulley 0.8m above A and supports a mass, M hanging freely. If a mass of 5kg is attached at B so as to keep the rod in a horizontal position, find the

i) value of M.

ii) reaction at the hinge.

68. The diagram below shows two uniform rods AB and BC of weights W and 3W respectively, which are smoothly hinged together at B. Point P is a point on AC which is vertically below B and AP = 4a, PC = 6a. The rods rest in equilibrium in vertical plane with the ends A and C on a smooth horizontal plane. The end C is connected to a point Q on AB by a light inelastic string.



Show that the magnitude of the reaction of the plane at A is $\frac{17}{16}$W and find the magnitude of the reaction of the plane at C. if BC = 10a and Q is the midpoint of AB, find the tension in CQ.

69. A non-uniform ladder AB of length 12m and mass 30kg has its Centre of gravity at the point of trisection of its length, nearer to A. the ladder rests with end A on the rough horizontal ground (coefficient of friction ¼) and B against a rough vertical wall (coefficient of friction $^1/_5$. The ladder makes an angle, θ with the horizontal such that tan θ = $\frac{9}{4}$. A straight horizontal string connects A to a point at the base of the wall vertically below B. what tension must the string be capable of withstanding if a man of mass 90kg is to reach the top of the ladder safely.

70. A non-uniform metallic beam AB of mass 30kg and length 4.4m balances with 45kg mass placed at end B. when support Q is placed 1.2m from B, find how far from end A the weight of the beam acts. If the beam balances again when an additional mass of 22.3kg is hang at end A and another support P is placed 0.8m from end A, determine the reactions at P and Q.